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THE KINETIC THEORY OF COMPTON SCATTERING BY RELATIVISTIC ELECTRONS

by

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ABSTRACT: Compton scattering by relativistic electrons, with induced scattering taken into consideration, is reviewed. The latter results in Compton reabsorption, limiting the brightness temperature of the scattered radiation. Compton reabsorption is shown to begin when brightness temperature is comparable to electron energy. Induced scattering results in intensification of the scattered radiation. Compton reabsorption source parameters are calculated. It is shown that if the cutoff in the infrared spectrum from pulsar NP 0532 is the result of Compton reabsorption, the angular dimensions of the radio, optical, and X-ray emission regions would be 10^{-16} rad (corresponding to linear dimensions of 1.2×10^{6} cm).

Introduction

The recent discovery of intense, compact sources has increased the interest taken in the processes involved in the Compton scattering of radiation by non-relativistic, as well as by relativistic, electrons. The scattering of low-frequency inter-galactic and relict radiation by relativistic electrons causes the relativistic electrons to lose a great deal of energy, and leads to the formation of X-rays [1, 2, 3]. References [4, 5, 6] investigated the Compton scattering of synchrotron radiation in compact, intense, radio radiation sources. The presence of intense radiation, and of high electron density, in quasars and pulsars, has stimulated the development of a theory of induced Compton scattering [7, 8, 9]. Reference [9] has developed a theory of induced scattering by relativistic electrons within the framework of an approximation of the Einstein coefficients. It is shown that when induced scattering is present, the brightness temperature of the scattered radiation cannot exceed some limit, determined by the mean energy of the electron. This paper reviews some of the questions of the kinetic theory of induced scattering. A precise /3*

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* Numbers in the margin indicate pagination in the foreign text.

expression for the Compton reabsorption factor is obtained, and the distortion in the scattered radiation spectrum is calculated. The results are applied directly to pulsar NP 0532.

I. Principal approximations and designations /4

Let $\varepsilon = h\nu$ and $\varepsilon_p = h\nu_p$ be the photon energy and frequency before and after scattering by an electron with energy E. We shall limit ourselves to the case of $E = \gamma^{-1} mc^2 \gg mc^2$ and $\frac{1}{4}\varepsilon\gamma^2 \leq \varepsilon_p \leq 4\varepsilon\gamma^{-2}$, and the Thomson approximation for the scattering cross section is valid. Radiation is unpolarized, and the distribution of relativistic electrons is isotropic. Then the cross section for the formation of a photon with energy ε_p, for scattering of a photon with energy ε by an electron with energy E can be found through [10].

$$\sigma(\varepsilon_p, \varepsilon, E) = \frac{\pi r_e^2}{4}\frac{\gamma^2}{\varepsilon}\left\{2\frac{\varepsilon_p}{\varepsilon}\gamma^2 - \frac{\varepsilon_p^2}{\varepsilon^2}\gamma^4 + 4\frac{\varepsilon_p}{\varepsilon}\gamma^2 \ln\frac{\varepsilon_p}{\varepsilon}\frac{\gamma^2}{4} + 8\right\} \qquad (1)$$

In what follows, we shall limit ourselves to the case of $\varepsilon_p \gg \varepsilon$, so that for conventional sources, photon densities will satisfy the inequality $n(\varepsilon_p) \ll n(\varepsilon)$. If N(E) is the energy density of the electrons, the kinetic equation for change in photon density will take the form

$$\frac{dn(\varepsilon_p)}{dt} = c\int n(\varepsilon)\sigma(\varepsilon_p, \varepsilon, E)N(E)dE\,d\varepsilon - \qquad (2)$$

$$- c\frac{(\pi\hbar c)^3}{\pi\varepsilon_p^2} n(\varepsilon_p)\int n(\varepsilon)\sigma(\varepsilon_p, \varepsilon, E)\varepsilon_p \frac{dN}{dE}dE\,d\varepsilon\,;$$

$$\frac{dn(\varepsilon)}{dt} = -c\int n(\varepsilon)\sigma(\varepsilon_p, \varepsilon, E)N(E)dE\,d\varepsilon_p + \qquad (3)$$ /5

$$+ c\cdot n(\varepsilon)\int\frac{(\pi\hbar c)^3}{\pi\varepsilon_p^2} n(\varepsilon_p)\sigma(\varepsilon_p, \varepsilon, E)\varepsilon_p\frac{dN}{dE}dE\,d\varepsilon_p\,.$$

The fact that electrons with energies $E + \varepsilon_p$ take part in the reverse processes of converting high-energy quanta into quanta of lesser energies

is taken into consideration. Only the term $\varepsilon_p \frac{dN}{dE}$ has been taken into consideration in the expansion by powers of the relativistic electron distribution function, because $\varepsilon_p \ll E$. The total number of photons is retained because

$$\frac{d}{dt} n = \frac{d}{dt} \left\{ \int n(\varepsilon_p) d\varepsilon_p + \int n(\varepsilon) d\varepsilon \right\} = 0 .$$

II. Properties of scattered radiation

Here, and in what follows, we shall limit ourselves to the power law of relativistic electron distribution, $N(E) = KE^{-n}$. Eq. (2), after integration, becomes (see appendix)

$$\frac{dn(\varepsilon_p)}{dt} = \frac{1}{4\pi} c \Phi(n) K \langle \varepsilon^{\frac{n-1}{2}} \rangle \langle \varepsilon \rangle^{-1} \rho \varepsilon_p^{-\frac{n+1}{2}} -$$

$$- c \frac{(\pi \hbar c)^3}{\pi} n(\varepsilon_p) \Phi(n+1) n K \langle \varepsilon^{\frac{n}{2}} \rangle \langle \varepsilon \rangle^{-1} \rho \varepsilon_p^{-\frac{n+4}{2}}$$

[] is the photon distribution mean, $\Phi(n)$ is found in [10], and is tabulated /6 in the appendix. The first term describes the spontaneous component of the scattered radiation, and plays a fundamental role in the non-intensive sources. The second describes the induced scattering, and leads to absorption. The scattering process in this particular approximation can be characterized by the coefficients of radiation and absorption (reabsorption). Thus, the Einstein coefficients method used in [9] is justified.

The emission coefficient can be written as follows in conventional designations

$$j(\nu_p) = \frac{1}{4\pi} \Phi(n) K \rho \langle \nu^{\frac{n-1}{2}} \rangle \langle \nu \rangle^{-1} \nu_p^{-\frac{n-1}{2}} , \qquad (4)$$

and the absorption coefficient can be written

$$\mu(\nu_p) = \frac{c^3}{8\pi} \Phi(n+1) n K \rho \langle \nu^{\frac{n}{2}} \rangle \langle \nu \rangle^{-1} \nu_p^{-\frac{n+4}{2}} . \qquad (5)$$

The transport equation for intensity of scattered radiation takes the conventional form

$$\frac{dJ(\nu_p)}{dL} = j(\nu_p) - \mu(\nu_p) J(\nu_p) . \qquad (6)$$

The solution to the latter is

$$J(\nu_p) = \frac{j(\nu_p)}{\mu(\nu_p)} \left(1 - e^{-\tau(\nu_p)} \right) = \qquad (7) \quad /7$$

$$= \chi(n) \frac{m}{n} \frac{\langle \nu^{\frac{n-1}{2}} \rangle}{\langle \nu^{\frac{n}{2}} \rangle} \nu_p^{5/2} \left(1 - e^{-\tau(\nu_p)} \right) .$$

Here ρ is the density of the scattered quanta, $\tau(\nu_p) = \varkappa L$ the optical thickness in terms of the Compton reabsorption, L is the scattering region. The magnitude $\varkappa(\nu_p)$ was obtained earlier in references [10, 11], and the approximate value of the reabsorption coefficient was first obtained in [9].

As will be seen from Eq. (7), there is a break in the spectrum in the low-frequency region in the scattered radiation observed. The spectral index for radiation from the optically dense regions is -5/2, and is close to quasiequilibrium, and is n-1/2 from the optically thin regions. The reabsorption frequency is found from the condition that $\tau(\nu_2) = 1$.

The angular dimensions θ are associated with the flux $F(\nu_2)$, observed at the reabsorption frequency

$$\nu_2 = \left[\frac{c^3}{8\pi}\Phi(n+1)n\langle\nu\rangle^{-1}\left\langle\nu^{\frac{n}{2}}\right\rangle\rho K L\right]^{\frac{2}{n+4}}, \qquad (8)$$

$$\theta = F^{\frac{1}{2}}(\nu_2)\,\nu_2^{-\frac{5}{4}}\left[0{,}5\,\frac{n\left\langle\nu^{\frac{n}{2}}\right\rangle}{m\chi(n)\left\langle\nu^{\frac{n-1}{2}}\right\rangle}\right]^{\frac{1}{2}}.$$

It is of interest to note the great similarity between the formulas /8
associated with Compton reabsorption and synchrotron reabsorption [3]. The latter will become clear if the synchrotron radiation is considered as the Compton scattering of virtual gyro-photons [13].

III. Scattered radiation

The transport equation for the intensity of the scattered radiation is obtained in the same way that Eq. (6) was obtained. Eq. (4) shows that we have the term $\beta(\nu)$, in addition to the term describing the spontaneous absorption that is not dependent on frequency. This term describes the amplification of the radiation attributable to the induced scattering

$$\frac{dJ(\nu)}{dL} = \varkappa(\nu) - \sigma_T N J(\nu) + \beta(\nu) J(\nu).$$

Here

$$\beta(\nu) = \int \frac{(\pi\hbar c)^3}{\pi\,\varepsilon_p^2} n(\varepsilon_p)\sigma(\varepsilon_p, \varepsilon, E)\varepsilon_p \frac{a\lambda}{aE}\, a E\, d\varepsilon_p =$$

$$= \frac{c^3}{8\pi}\Phi(n+1)\,nK\left\langle\nu_p^{-\frac{n+4}{2}}\right\rangle\left\langle\nu_p\right\rangle^{-1}\rho_p\,\nu^{\frac{n}{2}}.$$

Amplification is the result of the conversion of high-frequency quanta into low-frequency. Actually, when the density of the low-frequency quanta is much greater than that of the high-frequency, scattering with small change in frequency must be taken into consideration. This latter, evidently, will have a greater effect on the distortion of the low-frequency spectrum of the scattered radiation. /9

IV. Energy losses by relativistic electrons during induced Compton scattering

Let us consider for the calculation one electron with energy E, so that we can represent the electron distribution in the form of a delta function. Direct application of the cross section in the Eq. (1) form results in cumbersome calculations, so in what follows we shall limit ourselves to a cross section in the form

$$\sigma(\varepsilon_p, \varepsilon, E) = \sigma_T \delta\left(\varepsilon_p - \frac{4}{3}\langle\varepsilon\rangle\left(\frac{E}{mc^2}\right)^2\right).$$

Given these assumptions, and after multiplying Eq. (2) by ε_p, and integrating, we obtain an expression for losses

$$\frac{dE}{dt} = -\int \frac{dn(\varepsilon_p)}{dt}\varepsilon_p\, d\varepsilon_p = P_c - P_c \frac{c^2}{E}\frac{d}{d\nu_p}\left(\frac{J(\nu_p)}{\nu_p}\right). \qquad (9)$$

The first term describes the spontaneous energy losses [11] /10

$$P_c = -c\sigma_T p \frac{4}{3}\left(\frac{E}{mc^2}\right)^2.$$

The second term describes induced scattering losses. The derivative in the last term is taken for the frequency

$$\nu_p = \frac{4}{3}\langle\nu\rangle\left(\frac{E}{mc^2}\right)^2.$$

As will be seen from Eq. (9), relativistic electron energy losses in the self-absorption region are equal to zero if E = kT, where T is the brightness temperature of the radiation at frequency ν_p. If E < kT, the electrons acquire energy, and if E > kT, they lose energy. The electrons leave the self-absorption region almost without loss to the reverse Compton effect.

V. Nature of the optical and X-ray emission from pulsar NP 0532

Reference [14] has shown that it is possible to explain the optical and X-ray emissions from pulsar NP 0532 by the Compton scattering of low-frequency radio quanta by relativistic electrons. This postulates the coincidence of the radio, optical, and X-ray emission regions. Since the densities of the radio, optical and X-ray quanta are very high (the densities of the photons in the phase space in these bands are much greater than unity), the induced scattering process must be taken into consideration [14, 9]. The model of the /11 synchrotron nature of optical and X-ray emission [16], in which there is a free parameter for magnetic field intensity, divides the radio and optical emission regions because of synchrotron reabsorption, otherwise radio emissions would be absorbed. The model of the source with a Compton type of high-frequency radiation has no free parameters because the radio, optical, and X-ray emissions come from the same region, and are observed magnitudes. So, if it is assumed that the cutoff in the infrared band is the result of Compton reabsorption, using the necessary parameters [15], and Eq. (7), we obtain, unambiguously, the angular dimensions of the emission region, θ

$$\langle\nu\rangle = 2.5\cdot10^{7}\ \text{Hz};\quad F(\langle\nu\rangle) = 10^{-20}\ \text{erg/cm}^2\text{-sec-Hz}$$
$$\nu_2 = 7\cdot10^{14}\ \text{Hz};\quad F(\nu_2) = 10^{-25}\ \text{erg/cm}^2\text{-sec-Hz}$$
$$\theta = 10^{-16}\ \text{rad}$$

If we use the distance to the pulsar of 2 kpc, the linear dimensions of the emission region will be $L = 1.2\cdot10^{6}$ cm. Given these angular dimensions, the density of the radio radiation is $\rho = 10^{9}$ erg/cm^3, and the optical and X-ray radiation is $\rho_r = 6.7\cdot10^{12}$ erg/cm^3. When $n = 2$, the mean frequency of the scattered radiation is $\langle\nu\rangle = 3\cdot10^{16}$, and the mean energy of the relativistic electron is $E = 2\cdot10^{-2}$ erg. We obtain $K = 6\cdot10^{11}$ from Eq. (8), the equation for reabsorption frequency for these parameters. The electron density is 10^{13} to 10^{14} (the roughness of the latter can be estimated from the condition that outside the reabsorption region, $\nu > 10^{15}$ Hz, the scattering probability /12 is determined by the expression $\sigma_T N L \leq 10^{-5}$). The relativistic electron energy density and the power with which the relativistic electrons are injected, are $2\cdot10^{11}$ erg/cm^3, and 10^{34} erg/sec, respectively. During secondary Compton scattering (optical and X-ray quanta are converted into gamma quanta) the

electrons give up their energy completely to the photon because in this case the Thomson approximation is inoperable because the cross section is less than the Thomson by an order of magnitude. The mean frequency of the secondary scattered quanta is $\langle\nu_\gamma\rangle = 3\cdot10^{24}$ Hz, and the current at this frequency is $F(\langle\nu_\gamma\rangle = 2\cdot10^{-34}$ erg/cm^2-sec-Hz. This latter estimate does not contradict the observations [15]. Since most of the electron's energy loss occurs during secondary Compton scattering, the length of the electron's free path is 10^2 to 10^3 cm, and the presence of continuous injection uniformly distributed in the radiation region, at the sources must be assumed. In accordance with section III, electrons with energies $E < kT = 4\cdot10^{-2}$ erg can leave the radiation region almost without loss under predetermined conditions because of the presence of acceleration during induced Compton scattering.

The possibility of describing the high-frequency radiation from a pulsar coming from the region with specified angular dimensions and thickness of the order of the length of the electron's free path, encounters energy problems when interpreted in terms of hard gamma radiation.

In conclusion, we should point out certain difficulties associated with the suggested pulsar model. As noted in section III, the low-frequency radiation spectrum can be distorted quite a bit because of scattering with little change in frequency. This latter results in a non-relativistic limit to the /13
presence of the Bose condensation of photons (of the diffusion of photons at zero frequency) [17,18]. This difficulty obviously can be avoided if one takes a model in which the region of radio frequency radiation is larger than the region of optical and X-ray emissions. Now the density of the radio quanta will be much less and the scattering with small change in frequency can be ignored. We must, in addition, consider the anisotropy and heterogeneity in the distribution of electrons and photons. The latter, apparently, will reduce the energy difficulties associated with the presence of powerful gamma radiation.

APPENDIX

The following integrals will be taken for the calculation of the intensity of the scattered radiation, the reabsorption coefficient, and $\beta(\nu)$

$$\int_{E_{min}}^{\infty} \sigma(\varepsilon_p, \varepsilon, E)\, N(E)\, dE = \Phi(n)\, K\, \varepsilon_p^{-\frac{n+1}{2}}\, \varepsilon^{\frac{n-1}{2}}.$$

$$\int_{E_{min}}^{\infty} \sigma(\varepsilon_p, \varepsilon, E)\, \frac{dN}{dE}\, dE = \Phi(n+1)\, n \cdot K\, \varepsilon_p^{-\frac{n+2}{2}}\, \varepsilon^{\frac{n}{2}}$$

$$\int \sigma(\varepsilon_p, \varepsilon, E)\, d\varepsilon_p = \sigma_T = 6{,}65 \cdot 10^{-25}\ \text{см}^2.$$

Here $$E_{min} = mc^2 \left(\frac{\varepsilon_p}{4\varepsilon}\right)^{\frac{1}{2}},$$

$$\Phi(n) = 2^{-n}\, \frac{(n^2+4n+11)\,3}{(n+1)(n+3)^2(n+5)}\, \sigma_T\, (mc^2)^{1-n}.$$

are functions included in Eq. (8), and equal to

$$\frac{(n^2+4n+11)(n+2)(n+4)^2(n+6)}{(n^2+6n+16)(n+1)(n+3)^2(n+5)}.$$

The function will change from 3 to 1 in the interval $0 \leq n < \infty$.

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